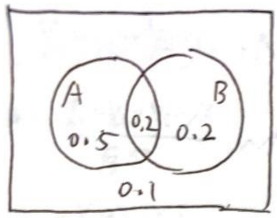
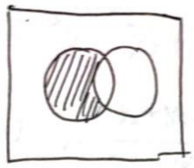
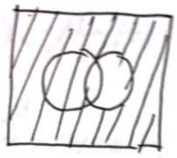
1、





$P(A-B)$



$P(A\cup\bar{B})$

$P(B\cdot\bar{A})=0.2$ ，$P(\bar{A})=0.3$， $\dfrac{P(B\cdot\bar{A})}{P(\bar{A})}=\dfrac{0.2}{0.3}=\dfrac{2}{3}$

2. $P(X=1)=\dfrac{\lambda^1 e^{-\lambda}}{1!}$ ，$P(X=2)=\dfrac{\lambda^2\cdot e^{-\lambda}}{2!}$ ，$P(X=1)=P(X=2)\Rightarrow\lambda=2$

则 $E(X)=\lambda=2$，$D(X)=\lambda=2=E(X^2)-[E(X)]^2\Rightarrow E(X^2)=6$

$P(X=0)=e^{-2}$，$P(X=1)=2e^{-2}$，$P(X=2)=2e^{-2}$，

$P(X=3)=\dfrac{2^3\cdot e^{-2}}{3!}=\dfrac{4}{3}e^{-2}$，$P(X=4)=\dfrac{2^4\cdot e^{-2}}{4!}=\dfrac{2}{3}e^{-2}$

$\therefore P(X\geq 4)=1-P(X=0)-P(X=1)-P(X=2)-P(X=3)=1-\dfrac{19}{3}e^{-2}$

恰有4人候车概率 $p=\dfrac{P(X=4)}{P(X\geq 4)}=\dfrac{2}{3e^2-19}$

3、记随机变量 X_i：第i次投所得点数．则 $E(X_i)=\dfrac{7}{2}$

$\therefore E(X)=E(X_1+X_2+X_3)=3E(X_1)=\dfrac{21}{2}$

$P(Y=2)=C_3^1\cdot\dfrac{1}{6}\cdot\left(\dfrac{4}{6}\right)^2+C_3^2\cdot\left(\dfrac{1}{6}\right)^2\cdot\dfrac{4}{6}+C_3^3\cdot\left(\dfrac{1}{6}\right)^3=\dfrac{61}{216}$

$P(Z=1)=C_3^1\cdot\dfrac{1}{6}\cdot\left(\dfrac{5}{6}\right)^2=\dfrac{25}{72}$，

$X=6$ 且 $Z=1$ 只有一种情况：1、2、3，$P(X=6,Z=1)=C_3^1\cdot\dfrac{1}{6}\cdot C_2^1\cdot\dfrac{1}{6}\cdot\dfrac{1}{6}=\dfrac{1}{36}$

$\therefore P(X=6|Z=1)=\dfrac{P(X=6,Z=1)}{P(Z=1)}=\dfrac{2}{25}$

4.(1). $X_1+X_2-X_3-\mu \sim N(0, 3\sigma^2)$, $X_2+X_3-X_4-\mu \sim N(0, 3\sigma^2)$.

则 $\dfrac{X_1+X_2-X_3-\mu}{\sqrt{3}\sigma} \sim N(0,1)$, $\dfrac{X_2+X_3-X_4-\mu}{\sqrt{3}\sigma} \sim N(0,1)$

$\therefore \left(\dfrac{X_1+X_2-X_3-\mu}{\sqrt{3}\sigma}\right)^2 \sim \chi^2(1)$, $\left(\dfrac{X_2+X_3-X_4-\mu}{\sqrt{3}\sigma}\right)^2 \sim \chi^2(1)$.

$\therefore \dfrac{\left(\dfrac{X_1+X_2-X_3-\mu}{\sqrt{3}\sigma}\right)^2}{\left(\dfrac{X_2+X_3-X_4-\mu}{\sqrt{3}\sigma}\right)^2} = \left(\dfrac{X_1+X_2-X_3-\mu}{X_2+X_3-X_4-\mu}\right)^2 \sim F(1,1)$

$\bar{X}-X_1 = \dfrac{1-n}{n}X_1 + \dfrac{1}{n}X_2 + \cdots + \dfrac{1}{n}X_n \sim N(0, \dfrac{n-1}{n}\sigma^2)$

$\therefore \dfrac{\bar{X}-X_1}{\sqrt{\frac{n-1}{n}}\sigma} \sim N(0,1)$, $\dfrac{(\bar{X}-X_1)^2}{\frac{n-1}{n}\sigma^2} \sim \chi^2(1)$

$\therefore$ $a = \dfrac{n-1}{n}\sigma^2$, $k=1$

(2). $S^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2 \xrightarrow{P} \sigma^2$, $n \to +\infty$

则 $\dfrac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2 = \dfrac{n-1}{n}S^2 \xrightarrow{P} \sigma^2$, $n \to +\infty$

(3). 构造检验统计量 $Z = \dfrac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \dfrac{6(\bar{X}-2)}{2} = 3(\bar{X}-2) \sim N(0,1)$,

$\because$ 左边检验

$\therefore P_- = P(Z \le Z_0) = P(Z \le -\frac{3}{2}) = \Phi(-\frac{3}{2})$

二.(1) 记事件A:天气好,则$\bar{A}$:天气不好;事件B:出门后半小时还未到甲地.

则由题知:$P(A)=0.8$, $P(\bar{A})=0.2$

$$P(B|A)=0.5, \quad P(B|\bar{A})=1$$

$\therefore P(B)=P(B|A)\cdot P(A)+P(B|\bar{A})\cdot P(\bar{A})=0.5\times 0.8+0.2=0.6$

(2) $P(A|B)=\dfrac{P(AB)}{P(B)}=\dfrac{P(B|A)\cdot P(A)}{P(B)}=\dfrac{0.5\times 0.8}{0.6}=\dfrac{2}{3}$.

三.(1) $\frac{1}{9}+\frac{3}{9}+\frac{2}{9}+\frac{2}{9}+a=1$, $\quad a=\frac{1}{9}$

(2) $E(XY)=P(XY=0)\cdot 0+P(XY=1)\cdot 1+P(XY=2)\cdot 2$

$=0+\frac{2}{9}\times 1+0\times 2=\frac{2}{9}$

$E(X)=P(X=0)\cdot 0+P(X=1)\cdot 1=0+\frac{5}{9}\times 1=\frac{5}{9}$

$E(Y)=P(Y=0)\cdot 0+P(Y=1)\cdot 1+P(Y=2)\cdot 2=0+\frac{4}{9}\times 1+\frac{1}{9}\times 2=\frac{2}{3}$

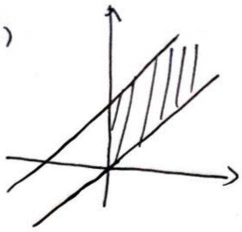
$\therefore Cov(X,Y)=E(XY)-E(X)\cdot E(Y)=\frac{2}{9}-\frac{5}{9}\times\frac{2}{3}=-\frac{4}{27}$

$\therefore X$与Y负相关

(3). $P(X=0|Y=1)=\frac{1}{2}$, $\quad P(X=1|Y=1)=\frac{1}{2}$

$$\therefore F_{X|Y}(x|1)=\begin{cases}0, & x<0\\ \frac{1}{2}, & 0\le x<1\\ 1, & 1\le x\end{cases}$$

四、(1)



$f_X(x)=\int_{-\infty}^{+\infty} f(x,y)dy=\int_{x}^{x+1} e^{-x}dy=e^{-x},\ x>0$

$f_Y(y)=\int_{-\infty}^{+\infty} f(x,y)dx=\begin{cases}\int_0^y e^{-x}dx=1-e^{-y},\ 0<y\le 1\\ \int_{y-1}^{y} e^{-x}dx=e^{1-y}-e^{-y},\ y>1\end{cases}$

$\therefore f_X(x)=\begin{cases}e^{-x},\ x>0\\ 0,\ x\le 0\end{cases}$ ，$f_Y(y)=\begin{cases}1-e^{-y},\ 0<y\le 1\\ e^{1-y}-e^{-y},\ y>1\\ 0,\ \text{其他}\end{cases}$

$\because f(x,y)=e^{-x}\ne f_X(x)\cdot f_Y(y)=e^{-x}\cdot(1-e^{-y})$ 或 $e^{-x}(e^{1-y}-e^{-y})$

$\therefore$ X与Y不独立

(2). 当 $y\le 1$ 时，$f_{X|Y}(x|y)=\dfrac{f(x,y)}{f_Y(y)}=\dfrac{e^{-x}}{1-e^{-y}}$.

$F_{X|Y}(x|y)=\int_0^x \dfrac{e^{-t}}{1-e^{-y}}dt=\dfrac{1-e^{-x}}{1-e^{-y}}$ ，$\therefore F_{X|Y}(X<0.5|Y=1)=\dfrac{1-e^{-\frac{1}{2}}}{1-e^{-1}}=\dfrac{\sqrt{e}}{\sqrt{e}+1}$

(3) $E(XY)=\iint_D xy\cdot f(x,y)dxdy=\int_0^{+\infty}dx\int_x^{x+1}xy\cdot e^{-x}dy=\dfrac{5}{2}$

五.(1). $E(X)=\int_0^\theta \frac{3}{\theta^3}x^3dx=\frac{3}{4}\theta=\mu_1$ $\therefore \hat{\theta}_1=\frac{4}{3}A_1=\frac{4}{3}\bar{X}$

$\because E(\bar{X})=\frac{3}{4}\theta$

$\therefore$由辛钦大数定律：$n\to+\infty$时，$\bar{X}\xrightarrow{P}\frac{3}{4}\theta$

$\therefore n\to+\infty$时，$\hat{\theta}_1\xrightarrow{P}\frac{4}{3}\cdot\frac{3}{4}\theta=\theta$，是相合估计.

(2). $E(X^2)=\int_0^\theta \frac{3}{\theta^3}x^4dx=\frac{3}{5}\theta^2$， $D(X)=E(X^2)-[E(X)]^2=\frac{3}{80}\theta^2$

由中心极限定理，$n\to+\infty$时 $\bar{X}$近 $N(\frac{3}{4}\theta,\ \frac{3}{80n}\theta^2)$

则 $\hat{\theta}_1=\frac{4}{3}\bar{X}\sim N(\theta,\ \frac{1}{15n}\theta^2)$

(3) $L(\theta)=3^n\cdot\theta^{-3n}\cdot\left(\prod_{i=1}^n x_i\right)^{2n}$，$0<X<\theta$，关于$\theta$单调递减.

则当θ取最小值，即$\max\{x_1\cdots x_n\}$时$L(\theta)$最大.

$\therefore \hat{\theta}_2=\max\{x_1\cdots x_n\}$

下求$\hat{\theta}_2$的分布.

$F(x)=\int_0^\theta\frac{3}{\theta^3}x^2dx=(\frac{x}{\theta})^3$，$0<x<\theta$

$\therefore F_{max}(x)=[F(x)]^n=\left(\frac{x}{\theta}\right)^{3n}$；$f_{max}(x)=\frac{3n}{\theta^{3n}}x^{3n-1}$，$0<x<\theta$

$\therefore E(X)=\int_0^\theta\frac{3n}{\theta^{3n-1}}x^{3n}dx=\frac{3n}{3n+1}\theta$

$\therefore E(\hat{\theta}_2)=\frac{3n}{3n+1}\theta\neq\theta$，不是无偏估计.

六、(1). 构造枢轴量 $F=\frac{S_1^2}{S_2^2}\Big/\frac{\sigma_1^2}{\sigma_2^2}\sim F(10,9)$.

则置信水平为95%的双侧置信区间：$\left[\frac{S_1^2}{S_2^2}\frac{1}{F_{0.025}(10,9)},\ \frac{S_1^2}{S_2^2}\frac{1}{F_{0.975}(10,9)}\right]$

$F_{0.025}(10,9)=3.96$, $F_{0.975}(10,9)=F_{0.025}(9,10)=3.78$

代入数据得：$[0.2697,\ 4.038]$

又∵1在该置区间内

∴在置信水平为0.95下，$\sigma_1^2=\sigma_2^2$

(2) 构造检验统计量：$T=\frac{(\overline{X}-\overline{Y})-(\mu_1-\mu_2)}{S_W\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\sim t(19)$

$S_W=\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}\approx 0.9548$

则拒绝域为：$\overline{X}-\overline{Y}<-t_{0.025}(19)\cdot S_W\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ 或 $\overline{X}-\overline{Y}>t_{0.025}(19)\cdot S_W\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$

而 $t_{0.025}(19)=2.093$

代入得：$\overline{X}-\overline{Y}<-0.8731$ 或 $\overline{X}-\overline{Y}>0.8731$

又∵$\overline{X}-\overline{Y}=0.2<0.8731$

∴在显著水平 $\alpha=0.05$ 下接受 H_0.